Privatization, Large Shareholders, and Sequential Auctions of Shares*

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Abstract

We study the government's decision to sell a state-owned enterprise to strategic investors in a common value auction setting. The government can choose to sell his control stake all at once, or to design a sequential auction of shares. The sequential auction allows information transmission, so that the winner of the first stake receives a signal about the value of control rights which will be sold at the second and final auction. We show that if bidders are symmetric, the sequential auction and the block auction are revenue equivalent. If instead one of the bidders has private information, the sequential auction is more profitable for the government. By disseminating information, the sequential auction forces the informed bidder to bid more aggressively, raising expected revenues.

1 Introduction

Consider a government seeking to privatize a state-owned enterprise. There is considerable uncertainty about the value of the company under private ownership, as it needs deep restructuring; domestic capital markets are thin and illiquid, and the company is too small to bear the cost of an international listing; furthermore, the government is running high budget deficits and under the pressure of international lending agencies, so that revenue generation is a priority of the sale.

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In this context, the government will choose an asset sale, auctioning his control stakes to domestic and possibly foreign strategic investors. This privatization strategy would certainly be more profitable than a heavily discounted fixed-price offering on domestic public equity markets. But how should the auction be designed to maximize proceeds?

Clearly, this problem is not only a theoretical puzzle, as asset sales are widely used in practice. In an in depth analysis of the choice of the privatization method, Megginson *et al.* [12] report that 1,225 of the 1,992 privatizations on public and private capital markets from 1977 through mid-1998 in 92 countries[1] are asset sales. Revenues from these sales are worth approximately US$222bn out of the US$719bn total including also privatizations on public equity markets. Furthermore, the privatization program of some countries (i.e. Mexico, see Lopez de Silanes [11]) relied almost entirely on asset sales.

Interestingly, when privatization occurs in private equity markets, governments auction off the majority ownership. The average asset sale has 75% of capital privatized, with a median value close to 100%. The average public offering has instead 39% of capital privatized, with a 27% median value (Megginson *et al.* [12]).

However, it is not completely clear whether divesting control in a block auction is always the most profitable strategy for the seller, especially when investors have asymmetric information. A well known result in auction theory with common values is that a bidder's small informational advantage amplifies "winner's curse" and reduces expected revenues, as the uninformed bidder is wary to beat a more informed competitor (Milgrom and Weber [14]). Under these circumstances, would it be better for the government to try to design the asset sale in order to attenuate informational asymmetries among the strategic investors?

In this paper, we provide a tentative answer to this question. We evaluate the performance in terms of revenue generation of an alternative auction method to sell control of state-owned enterprises by asset sale: the *sequential auction of shares*. In the sequential auction of shares, first a minority stake is sold, then the residual control rights are transferred at a second and final auction.

We assume the public value of the company under private ownership, i.e. the discounted value of its cash flow (taking risk factors into account), to be common knowledge; we instead introduce uncertainty over the common value of private benefits that can be extracted by controlling shareholders.

[1] The sources are *Privatisation International* and the *World Bank Privatization Database.*

The crucial feature of the sequential auction is that it allows for information transmission; the investor acquiring the minority stake receives a signal about the private benefits enjoyed if she will eventually control the company. Indeed, in our context the informational value even of a small stake is certainly non negligible.[2] The state-owned enterprise which is on sale is typically not listed, so reliable company information is not easily available to the public. A shareholder has instead full access to the books, participates in the meeting, and is in a better position to gauge the private benefits of control than a pure outsider.

The following results are obtained. If bidders are *ex ante* symmetrically uninformed about the value of control, then the sequential auction and the block auction of shares are perfectly equivalent in terms of proceeds, and the government is able to extract all the surplus in both auctions. The interim stage of the sequential auction generates an informational rent for the winner of the minority stake; but this rent is dissipated by competition at the first auction. The sequential auction is instead more profitable for the seller when one of the bidders has superior information which is gathered from "outside" the auction.

The intuition for this result is straightforward. In the block sale with asymmetric bidders, the informed investor wins very cheaply, obtaining a positive expected payoff. By choosing the sequential auction of shares, the government "levels the playing field", giving a chance to the uninformed investor to bid on an equal footing at the second auction. Indeed, the first auction may reduce the informational asymmetry, since the uninformed buyer may learn the value of the firm. The informed has now a greater incentive to bid aggressively for the first tranche of shares. By splitting the stake, the sequential auction enhances competition among the bidders, allowing the government to extract more surplus with respect to the block auction.

The result is obtained when the block and the sequential auction of shares are solved in the second-price sealed bid (Vickrey), and in first-price sealed bid format. However, the revenue differential becomes infinitesimal in the first-price format. The revenues of the block and sequential actions differ only by the (small) amount needed by the informed buyer to outbid the uninformed at the first auction, a strategy which allows to preserve a very large fraction of the informational rent.

[2] In the takeover context, Bradley, Desai, and Kim [1] show that buying toeholds in targets is a common strategy, and that substantial informational synergies may emerge in staggered corporate acquisitions.

The paper is organized as follows. In section 2, we review the literature. In section 3, we solve the model by considering Vickrey auctions of shares with symmetric and asymmetric bidders; in section 4, we solve the same models in the context of first-price sealed bid auctions. Section 5 considers a possible extension of the model. Section 6 concludes.

2 Related literature

In the privatization literature, a different theory of sequential sales has already been provided by Perotti [16], referring to government's inability to commit to future policy. If investors are uncertain about government's preferences, they are fearful to be expropriated *ex post* by populist politicians, who might interfere in the operating activities of the company to reallocate value to insiders. In this context, retaining a large residual passive stake signals that the government - as shareholder - is willing to bear residual risk, and not intended to interfere at the shareholders' expenses. If instead privatizing governments retain control, then strategic underpricing is necessary to signal commitment.

Cornelli and Li [5] analyze privatization schemes in the context of optimal auction design. When large shareholders differ in terms of the public value of the firm under their control and the private benefits they would extract from the company, the government faces a trade-off between revenues and efficiency. The highest bidder may not be the most efficient shareholder, as his high bid may reflect his high private benefits of control. Privatization schemes may be designed by the government to screen among investors with different plans, choosing the most efficient investor by use of the number of shares sold. It is shown that the government maximize his pay-off by making the allocation of shares contingent upon the bids, instead of committing to sell a fixed number of shares. This privatization mechanism allows to screen the most efficient investor by giving him a lower number of shares, so that the government ends up with a higher retained stake in a more valuable company.

Our model departs from these contributions in several ways; differently from Perotti, we do not model government preferences, so the issue of credibility remains unexplored. As to Cornelli and Li's paper, we do not assume that the control by different large shareholders entails different public values of the company and private benefits. In our context, clearly neither government'credibility nor revenues-efficiency trade-offs can be analyzed. On the other hand, we consider the implications of information transmission between the two tranches, which is a novel issue in the theoretical literature on privatization.

The analysis of multi-stage sales is certainly not new in corporate finance literature.

In particular, the idea that a two-stage sale could maximize the proceeds from the sale has been first developed by Zingales [19]. In his model, the value of the stake at the IPO stage is determined by who will control the firm eventually. If one of the rivals has a higher evaluation for the company, at the IPO stage dispersed shareholders with perfect foresight will anticipate the identity of the future controlling shareholder and will be willing to pay more for the company. By choosing optimally the size of the stake sold at the IPO, the two-stage sale allows the government to extract all the surplus from the rival and to generate higher total proceeds.

Albeit similar in spirit, our paper depart from Zingales' in two ways. First, we study the problem in an auction setting with incomplete information about the private benefits of control. Second, if we cast our model in the context of privatization on public equity markets, buying shares at IPO provides a signal about the value of the company, so that we allow information transmission between the two stages.

In a paper closely related to Zingales', Mello and Parson [13] study the optimal design of the sale to small and large shareholders, introducing secondary markets for shares. They find also find that an optimal selling mechanism is a sequential sale with an IPO followed by a block sale to the active investors at a discount, and concluding with a contingent sale of additional shares. In their model, a favourable treatment for potentially controlling shareholders maximizes the revenues from the sale as they assure an efficient ownership structure, which benefits all shareholders. This paper also models the IPO - which could be assimilated to our first tranche - as a mechanism for information transmission. We do not model different classes of investors, nor secondary tender offer markets, but focus instead on the private benefits enjoyed by only controlling shareholders. Indeed, shareholder become active with the prospect of enjoying some surplus which is not shared with other shareholders. This is probably the reason why blocks are typically traded at a premium, and not at a discount as predicted by Mello and Parson's model.

Our paper is also related to some contributions in auction theory.

Auctions of shares have been analyzed in a seminal paper by Wilson [18]. To our knowledge, he is the first to consider auctions of fractional shares of an object, such as a lease of tracts for oil and gas exploration and development. In his model, bidders submit a sealed tender with "demand" functions, or schedule of prices specifying the number of shares requested for each possible price. It is shown that an auction of shares may be severely dis-advantageous for the government: due to the manipulation by the bidders, he may experience a large reduction in revenue with respect to the unit auction. Our model differs from this paper by two features; first, our government is not forced to sell all the available supply of shares, but is allowed to end up owning a residual stake. Second, we do not analyze only the "public" value of a fraction of a an object, but - as conventional in the corporate finance literature - also the control rights attached to different stakes, and the associated private benefits.

Sequential auctions have been studied by Hausch [8], who first consider bidders encountering in two auctions for similar objects. In their model, the announced bids in the first auction convey information about the private value of the objects for sale. In line with Milgrom and Weber [14] result on information revelation in auctions, the expected revenue to the government is greater in the second stage than if there were no first stage and signals.

Branco [2] extends the setting incorporating synergies, namely complementarities among the different objects generating superadditive valuations. In a model with two "bundle" and two "unit" bidders and two objects for sale, it is shown that in the first auction a bundle bidder bids higher than his valuation. The prospect of enjoying the complementarities raises the competitive interest in getting the first object; bundle bidders will bid therefore aggressively in the first auction, while only the winner of the first aution will bid aggressively in the second auction, creating grounds for declining price over the sequence of auctions.

The main differences with these important contributions are the following. First, we consider sequential auctions for a *divisible* object, i.e. a company which is sold through a multiple issue of shares. Second, we do not allow for proper synergies between the two auctions. Indeed, the winner of the second stake enjoys the private benefits of control even if she did not acquire the first minority stake.

Bulow, Huang, and Klemperer [3] analyze toeholds in takeover battles. A "toehold" is a stake in a target company. It is shown that owning a toehold provides a strategic advantage in "common-value" takeover battles. Indeed, the toeholder can bid very aggressively since every price quoted represents a bid for the outstanding shares but also a ask for his stake. This behaviour increases the winner's curse so that he can win very cheaply. The target management can counteract this effect by "levelling the playing field", namely by giving a second bidder the opportunity to buy a toehold.

Although conceived in a takeover context, this model is quite close in the spirit to our common-value model, especially with respect to the idea of a second stake as a strategy to level the playing field. However, our key strategic variable is the private benefits of control, and not the public value of the company; furthermore, bidders do not have toeholds initially, but their buying toeholds emerges as an equilibrium outcome.

3 The model

Two risk-neutral bidders compete to acquire the control of a state-owned enterprise. To obtain it, bidder k (with $k = i, j$) must own a majority stake α. In this case, her holding will be worth $\alpha v + B$, where v is the public value of the firm (i.e. its expected present value, taking risk factors into account) and B are the private benefits enjoyed by the controlling shareholder. The private benefits of control have a common value B, which is a random variable taking values in $\Re_+$, with joint density and cumulative distribution $f(B)$ and $F(B)$, respectively.

Clearly, there might be private values components of the private benefits, stemming from synergies with the corporate assets of a particular large shareholder. But we believe that the common value assumption fits particularly well in the context of asset sales. Consider first the private benefits stemming from managerial opportunism and self-dealing. These corporate resources (perks, special dividends, excessive retained earnings, etc.) are up for grabs to whoever controls the company. And the expropriation of minority investors is limited by legal rules and by the quality of the enforcement, which varies across countries and should be independent from the identity of the controlling shareholder (see La Porta et al. [9]). Second, when bidders are foreign investors, these acquisitions often give the acquiring company enormous market power, as they can be used strategically to facilitate early entry. And the value of a large share of a foreign market should be similar for all bidders (see Cornelli and Li [5]).[3]

As in Cornelli and Li [5], we normalize to zero the value of the company under state's ownership, assuming that the government lacks the managerial skills to run the company

[3] We are not the first to apply the common value model to auctions of shares (see Bulow, Huang, and Klemperer [3]).

profitably. The company is small-medium sized so the government opts for an asset sale rather than an offering on public equity markets. The government is a profit-maximizer, so the sale is implemented through an auction.[4]

The government can choose to sell control auctioning α shares as a single block, or piecemeal through a sequence of two auctions for α_1 and α_2 shares, with $\alpha = \alpha_1 + \alpha_2$. We define the first method the *block auction* and the second the *sequential auction* of shares.

In the sequential auction of shares, the winner of the first auction receives a signal about the common value B: in particular, she learns the value with probability $p(\alpha_1)$ and with the complementary probability she does not learn anything. At the second auction, the winner of the first stake α_1 may be therefore better informed than the loser. One can realistically assume the probability of learning $p(\alpha_1)$ to be increasing in the value of the first stake. The larger the stake, the higher the power of the shareholder in the firm, and the higher the chances to know the real value of the private benefits of control.

By acquiring the first tranche, the bidder gathers information *overtly*, so that the loser knows to compete with a more informed bidder at the second stage.

Assumption 1 $\alpha_1 < \alpha_2$*: if different bidders win the two auctions, the winner of the second auction enjoys the private benefits of control.*

This assumption implies that when the government opts for the sequential auction, the largest shareholder will enjoy the private benefits of control. The first stake sold therefore represents a purely passive holding. If control rights were transferred at the first auction, then the block auction and the sequential auction would be qualitatively equivalent, and transmission of information could not take place.

Bids are denoted by b_k^h (with $h = 1, 2$, indicating the first or second stage of the sequential auction, respectively). The stakes are sold using a Vickrey (sealed-bid) second-price auction. We assume discrete bids with the smallest monetary unit fixed to ϵ (say, 1 dollar).

In what follows, we will solve the sequential auction identifying equilibrium strategies for the bidders, and then compare the expected revenues for the government with the revenues of the block sale. The pay-offs of the government are the revenue generated by one of the

[4]When there are several bidders, Bulow and Klemperer [4] show that expected revenue from a standard auction exceeds the expected revenue from bargaining with a predetermined buyer, even if the seller has all the bargaining power. See also Schmidt and Schnitzer [17].

auctions plus the value of the residual stake $(1-\alpha)$. The pay-offs of the bidders are given by the value of the holding minus the price paid for the shares.

We will first assume that bidders are *ex ante* symmetric and identically uninformed. Bidders can gather information only inside the sequential auction by acquiring the first stake. Then we will introduce asymmetries by assuming one bidder to have private information which is obtained outside the auction. In this part, we will solve completely both models in the simple context of second-price auctions.

3.1 Symmetric bidders

The game is solved by backward induction, so we start analyzing bidding strategies in the second auction for α_2 shares.

If bidder k has obtained the first tranche α_1 and if she has learnt the common value, the second tranche will be worth $\alpha_2 v + B$. If instead she has lost the first auction, or not learnt anything, she will be uninformed about the private benefits of control and the stake will be simply worth $\alpha_2 v + \int_0^\infty B f(B)\, dB = \alpha_2 v + EB$.

Lemma 1 *Suppose that bidder i has won the first auction and that she received the signal B. An equilibrium bidding strategy in the second auction is for the informed player i to bid $b_i^{2*}(B) = \alpha_2 v + B$, and for the uninformed player j to bid $b_j^{2*} = \alpha_2 v$.*

Proof. Suppose that the informed player deviates by bidding $b_i^2(B) > \alpha_2 v$. Then she wins the second auction and gets again a pay-off of B. Suppose that the informed player bids $b_i^2(B) < \alpha_2 v$. Then she loses the second auction and gets a pay-off of 0. Bidding her private evaluation is therefore an equilibrium strategy for the informed bidder. Suppose that the uninformed player deviates by bidding $b_j^2 > b_i^{2*}(B)$. Then she wins the second auction and gets a zero pay-off. Suppose now that the uninformed player deviates by bidding $b_j^2 < b_i^{2*}(B)$. Then she loses the second auction and gets a zero expected pay-off. Therefore bidding $b_j^2 = \alpha_2 v$ is a equilibrium strategy for the uniformed bidder. *Q.E.D.*

Lemma 2 *Suppose that bidder i has won the first auction and that she has not learnt anything about B. An equilibrium bidding strategy in the second auction is for either player to bid $b_k^{2*}(B) = \alpha_2 v + EB$, with $k = i, j$.*

Proof. Suppose that the informed player i deviates by bidding $b_i^2(B) > \alpha_2 v + EB$. Then she wins the second auction and gets a pay-off of zero. Suppose that the informed player deviates bids $b_i^2(B) < \alpha_2 v + EB$. Then she loses the second auction and gets a pay-off of zero. By symmetry, the same argument applies to the uninformed bidder. *Q.E.D.*

Having identified this equilibrium, we can compute the bidders' interim expected pay-offs. The informed bidder pay-offs EP_i^2 will be:

$$EP_i^2 = \int_0^\infty p(\alpha_1) B f(B)\, dB = p(\alpha_1) EB.$$

With probability $p(\alpha_1)$, bidder i knows the common value of the private benefits, while with probability $(1 - p(\alpha_1))$ the signal is completely uninformative. Being a second-price auction, in the first case the price paid for the shares by bidder i will be $\alpha_2 v$, while in the second case it will be $\alpha_2 v + EB$. At the interim stage, the expected pay-off is given by the expectation of the private benefits times the probability of learning the common value. The uninformed bidder's expected pay-off is clearly $EP_j^2 = 0$.

Having identified the equilibrium strategies and profits at the second stage, we can now turn to the first auction.

Lemma 3 *The symmetric equilibrium bidding strategy in the first auction is for either player to bid $b_i^{1*} = b_j^{1*} = \alpha_1 v + p(\alpha_1)EB$ and the equilibrium expected profits are zero for both bidders.*

Proof. Suppose that i deviates bidding $b_i^1 > \alpha_1 v + p(\alpha_1)EB$. Then she wins the first auction for shares and her expected payoffs will be $(\alpha_1 + \alpha_2)v + EB - \alpha_1 v - p(\alpha_1)EB - p(\alpha_1)\alpha_2 v - (1 - p(\alpha_1))(\alpha_2 v + EB) = 0$. Suppose instead that $b_i^1 < \alpha_1 v + EB$. Then she loses the first auction obtaining a zero pay-off. Therefore bidding b_i^{1*} in the first auction is an equilibrium strategy for bidder i. By symmetry, the same argument applies to bidder j. *Q.E.D*

Having identified the equilibrium bidding strategies, we can now compute the expected revenue of the government in the sequential auction and compare it with the revenue generated in the block auction.

The previous analysis has established that both bidders will bid the same amount $\alpha_1 v + p(\alpha_1)EB$ for the first stake. The winner of the first auction will also bid her private evaluation and pay a price equals to $\alpha_2 v$. When she does not learn anything, the price paid is simply $\alpha_2 v + EB$.

It is therefore straightforward to find the government's expected revenue of the sequential auction, ER_S. This is given by:

$$\begin{aligned} ER_S &= (1-\alpha_1-\alpha_2)v + \alpha_1 v + p(\alpha_1)EB \\ &+ p(\alpha_1)\alpha_2 v + (1-p(\alpha_1))(\alpha_2 v + EB) = v + EB. \end{aligned} \tag{1}$$

The first term in equation (1) represents the value of the residual stake owned by the government, while the other terms are the total proceeds from the two auctions.

Proposition 1 *The expected revenue from the sequential auction is equal to the expected revenue from the block auction.*

Proof. In the block auction, it is straightforward to prove that bidding $b = \alpha v + EB$ is an equilibrium strategy for both bidders. The bidders's expected profit is zero and the expected revenue for the government is $ER_B = (1-\alpha)v + \alpha v + EB = v + EB$. *Q.E.D.*

When bidders are symmetric, the two auctions are perfectly equivalent, and the government extracts all the surplus from the buyers. The informational rent generated at the interim stage of the sequential auction is dissipated by competition between bidders for the first stake.

3.2 Asymmetric bidders

In this part, one bidder is assumed to have private information. Information asymmetries are introduced as follows. Before participating in the sequential auction, both bidders are equally uninformed. However, bidder i receives a signal about the private benefits of control independently from the outcome of the first auction.[5] One of the two bidders might be an insider, who obtained confidential information from government officials, from advisors, or managers of the state-enterprise on sale.[6] Formally, with probability π she learns that

[5] Precisely, the informed bidder receives the signal after the first auction. This assumption allows us to neglect signalling issues.

[6] Privatization laws are often drafted to limit the risk of conflict of interests and collusion. For example, the managers of state-owned enterprises or staff of the privatization agency are often precluded to act as buyers. However, privatizations have been tarnished by allegations of corruption in many countries. See Guislain [6].

the private benefits are worth B; with the complementary probability, she does not learn anything. The uninformed bidder instead can obtain information only *inside* the auction, namely by acquiring the first α_1 shares. We also assume that the uninformed bidder knows her competitor to be privately informed.

As before, we first solve the sequential auction, and then compare the expected revenues of the sequential auction with the revenues of the block sale.

There are two cases to consider. If the privately informed bidder wins the first auction, she will receive two signals about B. One originates from being the holder of the α_1 shares, and another from her external pool of information. If instead the uninformed bidder wins the first auction, she will become informed, and both players will bid on equal footing at the second auction. Therefore the main difference with the symmetric case is that the uninformed bidder has now an opportunity to access the information of her opponent by acquiring the first α_1 shares.

Let us start by considering the first case. If the informed wins the first auction, we fall in the same situation described in the symmetric case, and equilibrium strategies are those identified in Lemma 1 and 2, resumed in Lemma 4.

Lemma 4 *Suppose that bidder i has won the first auction and that she has received the signal B. An equilibrium bidding strategy in the second auction is for player i to bid $b_i^{2*}(B) = \alpha_2 v + B$, and for player j to bid $b_j^{2*}(B) = \alpha_2 v$. Suppose instead that she has not learnt anything. Then equilibrium bidding strategy in the second auction is for either player to bid $b_k^{2*} = \alpha_2 v + EB$, with $k = i, j$.*

Proof. See Lemma 1 and 2.

The uninformed bidder's pay-offs of the second auction in the two sub-games will be equivalent to those identified in the symmetric case. The informed bidder's interim expected pay-offs will be instead

$$EP_i^2 = [\pi + (1 - \pi)p(\alpha_1)]EB, \tag{2}$$

where the term in brackets is the joint probability of learning B, as the informed bidder has a chance of learning it from her private signal, but also by winning the first auction. Clearly, the expected pay-offs will be higher with respect to the symmetric case, as now the buyer have two chances of learning the common value. The uninformed will obtain again a zero payoff.

Let us now turn to the second case. If the uninformed bidders wins the first auction, she receives a signal about B. As the bidders learn B with probability π and $p(\alpha)$ respectively, there are four cases to consider. Equilibrium strategies in the sub-games are stated in the following lemma, which is trivial to prove.

Lemma 5 *Suppose that bidder j has won the first auction;*

(i) If i and j have both learnt B, an equilibrium bidding strategy is for both players to bid $b_i^{2*}(B) = b_j^{2*}(B) = \alpha_2 v + B$.

(ii) If i has learnt B, and j has not learnt anything, an equilibrium bidding strategy is for i to bid $b_i^{2*}(B) = \alpha_2 v + B$, and for j to bid $b_j^{2*}(B) = \alpha_2 v$.

(iii) If j has learnt B, and i has not learnt anything, an equilibrium bidding strategy is for j to bid $b_j^{2*}(B) = \alpha_2 v + B$, and for i to bid $b_i^{2*}(B) = \alpha_2 v$.

(iv) If i and j have not learnt anything, an equilibrium bidding strategy is for both players to bid $b_i^{2*}(B) = b_j^{2*}(B) = \alpha_2 v + EB$.

The informed and uninformed bidder's EP_i^2 and EP_j^2 pay-offs will be, respectively:

$$EP_i^2 = \int_0^\infty \pi[1 - p(\alpha_1)]Bf(B)\,dB = \pi[1 - p(\alpha_1)]EB, \tag{3}$$

$$EP_j^2 = \int_0^\infty (1 - \pi)p(\alpha_1)Bf(B)\,dB = (1 - \pi)p(\alpha_1)EB. \tag{4}$$

It is easy to prove that the informed bidder's interim profits are higher in the case where she wins the first auction.[7] Obviously, this difference stems from from a higher probability of learning the common value when two signals are received.

Let us analyze bidders's behaviour in the first auction.

Lemma 6 *The equilibrium bidding strategy in the first auction is for the informed bidder i to bid:*

$$b_i^{1*} = \alpha_1 v + (1 - \pi)p(\alpha_1)EB + \epsilon,$$

and for the uninformed bidder j to bid

$$b_j^{1*} = \alpha_1 v + (1 - \pi)p(\alpha_1)EB.$$

[7] Indeed, $\pi + (1 - \pi)p(\alpha_1) > \pi(1 - p(\alpha_1))$, as $\pi - \pi p(\alpha_1) + p(\alpha_1) > \pi - \pi p(\alpha_1)$.

The equilibrium expected profits are given by:

$$EP_i^1 = \pi EB > 0$$

$$EP_j^1 = 0.$$

PROOF. Suppose that the informed player deviates by bidding $b_i^1 > b_i^{1*}$. Then she wins the first auction and gets the positive expected pay-off $EP_i^1 = (\alpha_1 + \alpha_2)v + EB - \alpha_1 v - (1-\pi)p(\alpha_1)EB - [\pi + (1-\pi)p(\alpha_1)]\alpha_2 v - [1 - \pi - (1-\pi)p(\alpha_1)](\alpha_2 v + EB) = \pi EB$. Suppose that the informed player deviates by bidding $b_i^1 < b_j^{1*}$. Then she loses the first auction and gets a expected pay-off $EP_i^2 = \pi(1 - p(\alpha_1))EB$, which is strictly lower than the pay-off when she wins both auctions. Bidding the equilibrium strategy is therefore an equilibrium strategy for the informed bidder. Suppose that the uninformed player deviates by bidding $b_j^1 > b_i^{1*}$. Then she wins the first auction and gets a pay-off

$$EP_j^1 = \alpha_1 v + (1-\pi)p(\alpha_1)EB - b_i^{1*} = -\epsilon.$$

Suppose now that the uninformed player deviates by bidding $b_j^1 < b_i^{1*}$. Then she loses both auctions and gets a zero expected pay-off. Therefore bidding b_j^{1*} is an equilibrium strategy for the uniformed bidder. *Q.E.D.*

We have therefore identified an equilibrium for the sequential auction with asymmetric bidders. We have now to compute the government's expected revenue and then compare it with the revenue of the block sale.

The expected revenue is given by the following expression:

$$\begin{aligned} ER_S &= (1-\alpha_1-\alpha_2)v + \alpha_1 v + (1-\pi)p(\alpha_1)EB + [\pi + (1-\pi)p(\alpha_1)]\alpha_2 v \\ &+ [1 - \pi - (1-\pi)p(\alpha_1)](\alpha_2 v + EB) = v + (1-\pi)EB. \end{aligned}$$

Proposition 2 *The expected revenue from the sequential auction is higher than expected revenue from the block auction.*

PROOF. In the block auction with asymmetric bidders, an equilibrium bidding strategy is for the informed to bid $\alpha v + B$, or $\alpha v + EB$, whether she learns or does not learn the value of the private benefits, and for the uninformed to bid αv. Therefore the expected revenue for the government is $ER_B = (1-\alpha)v + \alpha v = v$. Then $ER_S > ER_B$. *Q.E.D.*

Let us now contrast the equilibrium properties of the sequential action with the block auction of shares with asymmetric bidders. At equilibrium, the privately informed bidder wins both stages of the sequential auction, obtaining α as in the block auction. However, the surplus extracted by the informed investor in sequential auction is πEB, which is lower than the surplus in the block auction, EB.

Indeed, the informed bidder can win very cheaply the control rights of the company in the block auction. In the sequential auction instead, bidders compete *à la* Bertrand for the first stake. In this context, the informed investor has an incentive to outbid the opponent to get the higher expected profits when she remains the only informed bidder. The informed bidder faces the risk of losing her information advantage if the opponent wins the first auction; therefore she will bid aggressively at the first stage. This additional competitive pressure generated in the first stage of sequential auction is clearly beneficial to the government, who gets more expected revenues.

4 First-price auctions of shares

In what follows, we will change auction rules, and check if the result obtained in the context of Vickrey auctions still holds in first-price sealed-bid format. As before, we will solve the (first-price) sequential auction, and compare the expected revenues for the government with those generated by the (first-price) block auction. The model is solved again with symmetric and asymmetric bidders.

4.1 Symmetric bidders

The game is solved by backward induction, so we start by analyzing bidding strategies in the second auction for α_2 shares.

As before, if bidder k has obtained the first stake α_1 and if she has learnt B, we fall in a particular case of the asymmetric common value auction studied by Milgrom and Weber [15]. In this context, one of the bidders observes no information, therefore her (mixed) bidding strategy can be described by a probability distribution G over $\Re_+$, representing a random choice of a bid. The informed bidder should instead play a pure strategy, which is a function mapping her signal into the domain of non-negative bids.

Theorem 1 (Milgrom and Weber, [15]) *Suppose that bidder i has won the first auction* *and that she received the signal B. The unique equilibrium strategies are for player i to bid $b_i^{2*}(B)$, and for the uninformed player j to bid according to $G^*(b)$, where:*

$$\begin{aligned} b_i^{2*}(B) &= \alpha_2 v + B - \frac{\int_0^B F(s)\,ds}{F(B)}, & (5)\\ G^*(b) &= F(b_i^{2*-1}(b)). & (6)\end{aligned}$$

At equilibrium, the distribution of bids is the same for both bidders.

PROOF. See Appendix.

If instead bidder k has obtained the first stake α_1 but has not learnt anything, equilibrium strategies are the same identified in Lemma 2, resumed in the following Lemma.

Lemma 7 *Suppose that bidder i has won the first auction and that she has not learnt anything about B. An equilibrium bidding strategy in the second auction is for either player to bid $b_k^{2*}(B) = \alpha_2 v + EB$, with $k = i, j$.*

PROOF. See Lemma 2.

Having identified this equilibrium, we can compute the bidders' interim expected pay-off. The informed bidder pay-offs EP_i^2 will be:

$$\begin{aligned} EP_i^2 &= \int_0^\infty \left\{ p(\alpha_1) \left[\alpha_2 v + B - b_i^{2*}(B) \right] F(B) \right\} f(B)\,dB \\ &= p(\alpha_1) \int_0^\infty \int_0^B F(s)\,ds f(B)\,dB. \end{aligned}$$

By Fubini theorem,

$$EP_i^2 = p(\alpha_1) \int_0^\infty (1 - F(B))F(B)\,dB. \tag{7}$$

When the informed bidder observes the signal B and makes her equilibrium bid, he wins with probability $G^*(b)$. By Theorem 1, this is equal to $F(B)$. When he wins, she receives a stake worth $\alpha_2 v + B$, but pays only $b_i^{2*}(B)$. Obviously, the expression in brackets has to be integrated for all possible values of B. When instead the informed does not learn anything, her expected payoffs will be zero.

The uninformed bidder's expected pay-off is clearly $EP_j^2 = 0$. When the informed learns B, the uninformed plays a mixed strategy which is identical to the distribution of the bids of the informed. As both strategies have full support, the expected profit for the uninformed bidder is zero (see Appendix). Profits are also zero when the opponent does not learn anything, as both will bid the same amount.

Having identified the equilibrium strategies and profits at the second stage, we can now turn to the first auction.

Lemma 8 *The symmetric equilibrium bidding strategy in the first auction is for either player to bid*

$$b_i^{1*} = b_j^{1*} = \alpha_1 v + p(\alpha_1) \int_0^{\infty} (1 - F(B))F(B)\, dB,$$

and equilibrium expected profits are zero for both bidders.

Proof. Suppose that i deviates bidding $b_i^1 > b_i^{1*}$. Then she wins the first auction for shares, but obtains a negative pay-off, as the expected profits of the second auction are as in equation (7). Suppose instead that $b_i^1 < b_i^{1*}$. Then she loses the first auction obtaining a zero pay-off. Therefore bidding b_i^{1*} in the first auction is an equilibrium strategy for bidder i. By symmetry, the same argument applies to bidder j. *Q.E.D*

Having identified equilibrium strategies, we can compute the expected revenue of the government ER_S in the first-price sequential auction. This is given by the following expression:

$$\begin{aligned} ER_S &= (1 - \alpha_1 - \alpha_2)v + \alpha_1 v + p(\alpha_1) \int_0^{\infty} (1 - F(B))F(B)\, dB \\ &+ p(\alpha_1) \int_0^{\infty} b_i^{2*}(B) 2F(B) f(B)\, dB \\ &+ (1 - p(\alpha_1))(\alpha_2 v + EB) \end{aligned} \tag{8}$$

The first term in equation (8) represents the value of the residual stake owned by the government; the second and the third term are simply the optimal bid in the first auction. The last two terms are instead the expected price paid at the second auction. With probability $p(\alpha_1)$, players are asymmetrically informed, and the revenue will be given by the expected value of the first-order statistic of their bids. With probability $(1 - p(\alpha_1))$, players bid on equal footing the value of the second stake plus the expected value of the private benefits.

Inserting the optimal value of the bid in the second auction b_i^{2*} in (8) yields

$$ER_S = v + EB + p(\alpha_1)\left[\int_0^\infty 2BF(B)f(B)\,dB - \int_0^\infty (1-F(B))F(B)\,dB - EB\right]$$

Proposition 3 *The expected revenue from the sequential auction is equal to the expected revenue from the block auction.*

Proof. In the first-price block auction with symmetric bidders, the expected revenue for the government is clearly $ER_B = v + EB$. We have to prove that

$$\int_0^\infty 2BF(B)f(B)\,dB = \int_0^\infty (1-F(B))F(B)\,dB + EB. \tag{9}$$

By integrating by parts the expected value, we obtain

$$\int_0^\infty Bf(B)\,dB = -B(1-F(B))|_o^\infty + \int_0^\infty (1-F(B))\,dB.$$

The right hand side of equation (10) becomes

$$\int_0^\infty (1-F(B))F(B)\,dB + \int_0^\infty (1-F(B))\,dB = \int_0^\infty (1-F(B)^2)\,dB$$

Integrating it by parts again yields

$$\int_0^\infty (1-F(B)^2)\,dB = B(1-F(B)^2)|_o^\infty + \int_0^\infty 2BF(B)f(B)\,dB$$

The first term of the right hand side is zero, which gives the identity (10). *Q.E.D.*

With symmetric bidders, we have therefore established revenue equivalence of the first-price and second-price sealed-bid auctions of shares. The government extracts all the surplus from the bidders in both auction formats.

4.2 Asymmetric bidders

Again, there are two cases to consider. If the privately informed bidder wins the first auction, she will receive two signals about B. One originates from being the holder of the α_1 shares, and another from her external pool of information. If instead the uninformed bidder wins the first auction, she will become informed, and both players will bid on equal footing at the second auction.

As before, we start by finding equilibrium strategies when the privately informed bidder wins the first auction, receiving two signals about B. If this case, we fall in the same situation described in the symmetric setting, and equilibrium strategies are those identified in the following lemma.

Lemma 9 *Suppose that bidder i has won the first auction and that she has received the signal B. Then equilibrium bidding strategies in the second auction are those identified in Theorem 1. Suppose instead that she has not learnt anything. Then equilibrium bidding strategy in the second auction is for either player to bid $b_k^{2*} = \alpha_2 v + EB$, with $k = i, j$.*

PROOF. See Theorem 1 and Lemma 7.

The interim informed bidder's pay-offs EP_i^2 are given by

$$\begin{aligned} EP_i^2 &= \int_0^\infty \left\{ [\pi + (1-\pi)p(\alpha_1)] \left[\alpha_2 v + B - b_i^{2*}(B) \right] F(B) \right\} f(B)\, dB \\ &= [\pi + (1-\pi)p(\alpha_1)] \int_0^\infty (1 - F(B)) F(B)\, dB \end{aligned}$$

where the term in brackets is the joint probability of learning B. Clearly, the uninformed bidder will obtain a zero interim expected payoff.

Let us now turn to the second case, where the uninformed bidders wins the first auction and receives a signal about B. Recall that the informed buyer learns B with probability π, while the uninformed with probability $p(\alpha)$. SO there are four cases to consider. Equilibrium strategies in this sub-game are stated in the following lemma, which is trivial to prove.

Lemma 10 *Suppose that bidder j has won the first auction;*

(i) If i and j have both learnt B, an equilibrium bidding strategy is for both players to bid $b_i^{2*}(B) = b_j^{2*}(B) = \alpha_2 v + B$.

(ii) If i has learnt B, and j has not learnt anything, the equilibrium bidding strategies are those identified in Theorem 1.

(iii) If j has learnt B, and i has not learnt anything, the equilibrium bidding strategies are symmetric to those identified in Theorem 1.

(iv) If i and j have not learnt anything, an equilibrium bidding strategy is for either player to bid $b_k^{2*} = \alpha_2 v + EB$, with $k = i, j$.

The bidders' interim expected pay-offs of the second auction EP_i^2 and EP_j^2 pay-offs will be:

$$
\begin{aligned}
EP_i^2 &= \int_0^\infty \left\{\pi(1-p(\alpha_1))\left[\alpha_2 v + B - b_i^{2*}(B)\right]F(B)\right\} f(B)\,dB \\
&= \pi(1-p(\alpha_1))\int_0^\infty (1-F(B))F(B)\,dB \\
EP_j^2 &= \int_0^\infty \left\{(1-\pi)p(\alpha_1)\left[\alpha_2 v + B - b_j^{2*}(B)\right]F(B)\right\} f(B)\,dB \\
&= (1-\pi)p(\alpha_1)\int_0^\infty (1-F(B))F(B)\,dB
\end{aligned}
$$

The same proof that we used to show that the informed bidder's interim profits are higher in the case where she wins the first auction in the second-price format can be applied here (see footnote 7).

Let us analyze bidders's behaviour in the first auction.

Lemma 11 *The equilibrium bidding strategy in the first auction is for the informed bidder i to bid:*

$$b_i^{1*} = \alpha_1 v + (1-\pi)p(\alpha_1)\int_0^\infty (1-F(B))F(B)\,dB + \epsilon,$$

and for the uninformed bidder j to bid

$$b_j^{1*} = \alpha_1 v + (1-\pi)p(\alpha_1)\int_0^\infty (1-F(B))F(B)\,dB.$$

The equilibrium expected profits are given by:

$$EP_i^1 = \pi\int_0^\infty (1-F(B))F(B)\,dB - \epsilon$$

$$EP_j^1 = 0.$$

Proof. Suppose that the informed player deviates by bidding $b_i^1 = b_i^{1*} + \epsilon$. Then she wins the first auction and gets the expected pay-off

$$
\begin{aligned}
EP_i^1 &= \alpha_1 v + [\pi + (1-\pi)p(\alpha_1)]\int_0^\infty (1-F(B))F(B)\,dB - \alpha_1 v \\
&- (1-\pi)p(\alpha_1)\int_0^\infty (1-F(B))F(B)\,dB - 2\epsilon \\
&= \pi\int_0^\infty (1-F(B))F(B)\,dB - 2\epsilon.
\end{aligned}
$$

Suppose that the informed player deviates by bidding $b_i^1 < b_j^{1*}$. Then she loses the first auction and gets the expected pay-off

$$EP_i^2 = \pi(1 - p(\alpha_1)) \int_0^\infty (1 - F(B))F(B)\, dB,$$

which is strictly lower than the pay-off when she wins the first auctions. Bidding the equilibrium strategy is therefore an equilibrium strategy for the informed bidder.

Suppose that the uninformed player deviates by bidding $b_j^1 = b_i^{1*} + \epsilon$. Then she wins the first auction and gets a pay-off

$$\begin{aligned} EP_j^1 &= \alpha_1 v + (1-\pi)p(\alpha_1) \int_0^\infty (1 - F(B))F(B)\, dB - \alpha_1 v \\ &- (1-\pi)p(\alpha_1) \int_0^\infty (1 - F(B))F(B)\, dB - 2\epsilon = -2\epsilon. \end{aligned}$$

Suppose now that the uninformed player deviates by bidding $b_j^1 < b_j^{1*}$. Then she loses the first auction and gets a zero expected pay-off. Therefore bidding b_j^{1*} is an euqilibrium strategy for the uniformed bidder. *Q.E.D.*

We have therefore identified the (unique) equilibrium for the first-price sequential auction with asymmetric bidders. We have now to compute the government's expected revenue and then compare it with the revenue of the block sale. The expected revenue is given by the following expression:

$$\begin{aligned} ER_S &= (1 - \alpha_1 - \alpha_2)v + \alpha_1 v + (1-\pi)p(\alpha_1) \int_0^\infty (1 - F(B))F(B)\, dB \\ &+ [\pi + (1-\pi)p(\alpha_1)] \int_0^\infty b_i^{2*}(B) 2F(B)f(B)\, dB \qquad (10) \\ &+ (1-\pi)(1 - p(\alpha_1)(\alpha_2 v + EB) + \epsilon \end{aligned}$$

Plugging the informed bidder's optimal bid in the second auction gives

$$\begin{aligned} ER_S &= v + (1-\pi)p(\alpha_1) \int_0^\infty (1 - F(B))F(B)\, dB + (1-\pi)(1 - p(\alpha_1)EB \\ &+ [\pi + (1-\pi)p(\alpha_1)] \left\{ \int_0^\infty 2BF(B)f(B)\, dB - 2\int_0^\infty (1 - F(B))F(B)\, dB \right\} + \epsilon \end{aligned}$$

Using equation (2.10) established in Proposition 3, we get

$$ER_S = v + EB - \pi \int_0^\infty (1 - F(B))F(B)\, dB + \epsilon$$

Proposition 4 *The expected revenue from the sequential auction is higher than expected revenue from the block auction.*

Proof. In the block auction with asymmetric bidders, the expected revenue for the government is

$$\begin{aligned} ER_B &= (1-\alpha)v + \alpha v + \pi \int_0^\infty b_i^{2*}(B) 2F(B) f(B)\, dB + (1-\pi)(\alpha v + EB) \\ &= v + EB - \pi \int_0^\infty (1 - F(B)) F(B)\, dB \end{aligned}$$

Then $ER_S > ER_B$. *Q.E.D.*

The difference between the expected revenues of the government in the sealed-bid first price block and sequential auctions is still positive, but it shrinks to ϵ, the small but - by assumption - strictly positive amount through which the informed investor outbids the opponent in the first-stage of the sequential auction.

5 An extension: auctions of shares with entry

As we mentioned in the introduction, the auction setting is particularly realistic in privatization by asset sale. In this context, the divesting government posts an invitation to tender for a given number of shares on the Privatization Agency website or international financial press; then perspective bidders spend some time evaluating the company, drafting the documents for the pre-qualification stage (if any), and finally preparing and sending out their bids. Participation in an auction of share might involve costs. Does the presence of sunk costs affect entry decision, equilibrium bidding behaviour, and importantly the government's expected revenue?

In this section, we explore this possibility, and assume that participating in an auction entails each bidder paying a cost c before the auction. For simplicity, we will restrict to the analysis of second-price sealed-bid auctions of shares.[8]

Let us start from the simple case when bidders are symmetrically informed *ex ante*. If one bidder learns B after obtaining the first tranche, then an equilibrium bidding strategy for the uninformed is not to enter. If the uninformed bidder enters, her net profit from the second auction will be $-c$, as she will not be able to recover the entry costs if she loses the auction. When the uninformed does not enter, the auction becomes a bargaining

[8] To our knowledge, the asymmetric common value auction with entry has not already analyzed in the literature, and solving it goes beyond the scope of this extension. For the symmetric model, see Levin and Smith [10].

game between the government and the large shareholder with private information. In this context, we can apply the Coase conjecture, stating that the price will converge to the lowest possible valuation of the buyer, namely $\alpha_2 v$, with the buyer extracting all the surplus (Gul, Sonnenschein, and Wilson [7]). If instead the winner of the first tranche did not learn anything, then both bidders enter and bid $\alpha_2 v + EB - c$. In this equilibrium, the informed bidder's interim expected profits will be $p(\alpha_1)EB$, and the uninformed gets zero.[9]

Both large shareholders will therefore bid $\alpha_1 v + p(\alpha_1)EB - c$ at the first auction, obtaining again $(\alpha_1 + \alpha_2)v + EB - \alpha_1 v - p(\alpha_1)EB + c - p(\alpha_1)\alpha_2 v - (1 - p(\alpha_1))(\alpha_2 v + EB - c) - c - (1 - p(\alpha_1))c = 0$.

The expected revenues of the sequential auction are:

$$\begin{aligned} ER_S &= (1 - \alpha_1 - \alpha_2)v + \alpha_1 v + p(\alpha_1)EB - c + p(\alpha_1)\alpha_2 v + (1 - p(\alpha_1))(\alpha_2 v + EB - c) \\ &= v + EB - c - (1 - p(\alpha_1))c. \end{aligned}$$

Revenues of block auction will be $ER_B = (1 - \alpha)v + \alpha v + EB - c = v + EB - c$. In the symmetric case without entry costs, we proved revenue equivalence between the sequential and the block auction. Now with entry costs the block auction is always more profitable for the seller. Indeed, the bidders in the sequential auction will discount on the bids the additional costs of the second auction.

Let us analyze the more interesting case when bidders are asymmetrically informed, starting from the second auction. If the informed bidder wins the first auction, then her interim expected profits have simply to be adjusted for the joint probability of learning B, and are equal to $[\pi + (1 - \pi)p(\alpha_1)]EB$, and remain zero for the uninformed.

If instead the uninformed wins the first auction, we have the four cases identified in Lemma 5. When one of the two bidders does not learn B (case (ii) and (iii)), the auction becomes the bargaining game that we analyzed in the symmetric case. When both learn or do not learn B (case (i) and (iv)), the symmetric equilibrium bidding strategies are $\alpha_2 v + B - c$ and $\alpha_2 v + EB - c$, respectively. Equilibrium interim profits are therefore $\pi(1 - p(\alpha_1)EB$ and $(1 - \pi)p(\alpha_1)EB$ for the informed and the uninformed, respectively.

At the first auction, bidders will play the same Bertrand game analyzed in Lemma 6, adjusting bids to take into account participation costs: the informed outbids by an ϵ the

[9] We are assuming that entry costs are zero when there is bargaining instead of the auction. However, the assumption of positive bargaining costs does not affect the qualitatively our results.

equilibrium strategy of the uninformed, i.e. $b_j^{1*} = \alpha_1 v + (1-\pi)p(\alpha_1)EB - c$.

The informed bidder's *ex ante* profits are:

$$\begin{aligned} EP_i^1 &= (\alpha_1+\alpha_2)v + EB - \alpha_1 v - (1-\pi)p(\alpha_1)EB + c - [\pi + (1-\pi)p(\alpha_1)]\alpha_2 v \\ &- [1-\pi-(1-\pi)p(\alpha_1)](\alpha_2 v + EB - c) - c - [1-\pi-(1-\pi)p(\alpha_1)]c = \pi EB. \end{aligned}$$

The bidders' expected profits are therefore unaffected by participation costs. However, the expected revenues of the government are different in the model with entry costs. The revenues of the sequential auction with entry are given by the following expression:

$$\begin{aligned} ER_S &= (1-\alpha_1-\alpha_2)v + \alpha_1 v + (1-\pi)p(\alpha_1)EB - c + [\pi + (1-\pi)p(\alpha_1)]\alpha_2 v \\ &+ [1-\pi-(1-\pi)p(\alpha_1)](\alpha_2 v + EB - c) = v + (1-\pi)EB - c - (1-\pi)c. \end{aligned}$$

As rational bidders discount participation costs in their bids, the expected revenues will be lower with respect to the sequential auction with free entry. In particular, both bidders will pay c and enter the first auction; bidders will instead bear participation costs of the second auction with probability $(1-\pi)$, i.e. the probability that the informed does not learn the private value. With the complementary probability, the auction becomes a bargaining game.

Let us now consider the block auction with entry costs. When bidders are asymmetric *ex ante*, the uninformed will not enter the auction for α shares. The privately informed large shareholder will bargain with the government, extracting all the surplus. The expected revenue for the government are simply v.

By comparing the expected revenues, we can observe that the sequential auction allows the government to retain a fraction of the informational surplus which is decreasing with the precision of the signal of the informed bidder. However, the sequential auction involves entry costs, which are discounted in the bids. This trade-off can be easily solved: as long as $EB > \frac{2c-\pi c}{1-\pi}$, the sequential auction is more profitable for the seller. This condition is certainly met when transaction costs are negligible with respect to the economic value of private benefits of control, as it often occurs in practice.

6 Concluding remarks

In this paper, we have tried to provide a rationale based on revenue maximization for sequential sales of shares as a method to divest the control of state-owned enterprises. The sequential auction tends to generate higher expected revenue for the government, forcing bidders to compete twice to obtain the control rights of the company. Our result critically hinges upon the assumption of information asymmetries about the value of the company among the bidders, and to the possibility of learning that value by acquiring minority stakes.

The result is obtained in a very simple setting with only two bidders, and for a structure of signals which is far from general. So it could be interesting to see whether it will still hold in a model with n bidders, and for generic signals. It would also be natural to try to extend the model in the context of privatization on public equity markets, where sequential issues could also be used a learning device.

Although very simple and stylized, the model yields an empirical implication: the aggregate revenues from sequential transfers of control should be higher where information asymmetries among potential investors are larger. Ongoing research is trying to bring this implication to the data.

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A Appendix

Proof of Theorem 1. The expected utility for the informed bidder is $(B-b)G(b)$, hence the F.O.C.

$$B = b + \frac{G(b)}{g(b)}, \tag{11}$$

which defines the inverse $b^{-1}(b)$ of the bid function $b(B)$.[10] The uninformed bidder wins with a bid p if $p > b(B)$, or if $B < b^{-1}(p)$; hence, her expected utility is

$$\pi(p) = \int_0^{b^{-1}(p)} (B - p) f(B)\, dB.$$

The uninformed bidder plays a mixed strategy. Therefore, she must be indifferent between all the elements of the support of her strategy, i.e. $\frac{d\pi(p)}{dp} = 0$, or

$$b^{-1'}(p)(b^{-1}(p) - p) f(b^{-1}(p)) - \int_0^{b^{-1}(p)} f(B)\, dB = 0.$$

By noting that $p = b(B)$ and $b^{-1'}(p) = \frac{1}{b'(B)}$, the equation can be integrated to

$$b(B) = \frac{\int_0^B s f(s)\, ds}{F(B)} - \frac{k}{F(B)}.$$

The only solution compatible with nonnegative profits for the uninformed and with the optimality of the strategy is $k = 0$, hence

$$b(B) = B - \frac{\int_0^B F(s)\, ds}{F(B)}. \tag{12}$$

Inserting (11) in (12) gives the $G(.)$ distribution. Indeed,

$$\frac{G(b(B))}{g(b(B))} = \frac{\int_0^B F(s)\, ds}{F(B)},$$

or

$$\frac{b'(B) g(b(B))}{G(b(B))} = \frac{b'(B) F(B)}{\int_0^B F(s)\, ds} = \frac{f(B)}{F(B)},$$

and $G(b(B)) = F(B)$. *Q.E.D.*

[10] Subscripts and superscripts are suppressed for notational convenience.