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TRIALS AND ERRORS: PLEA BARGAINING AS A LEARNING DEVICE

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B E L G I U M

TRIALS AND ERRORS:
PLEA BARGAINING AS A LEARNING DEVICE

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Abstract

The plea bargaining procedure, namely the viability of a stage of bargaining between prosecutor and defendant in criminal suits, is analyzed in the framework of a two-sided incomplete information game. It is shown that, for a given parameter configuration, there exists a Bayesian equilibrium with perfect screening of the guilty defendant. In the repeated game, a prosecutor who systematically resorts to the informative strategy and updates her beliefs in a Bayesian fashion asymptotically learns the "truth" in terms of proportion of guilty parties in the whole population of the indictees.

Keywords: Plea Bargain, Litigation, Bayesian Learning.

JEL classification: C72, D83, K41.

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1 Introduction

In the United States, 86 % of 1990 filings for federal crimes has been resolved via extra-judicial procedures; in 1988, in the 75 major counties, 95 % of the convictions for crimes punished with non-pecuniary sanctions resulted from a negotiated plea of guilty.[1] Empirical analyses[2] and anedoctical evidence show that a large proportion of antitrust violations are settled at pre-trial both at FTC and at the General Division for Competition of the European Community where resolutions are rendered. Many European countries in which the *civil law* tradition[3] is deeply rooted are reforming their criminal codes introducing plea bargains to reduce the backlog that stymie the administration of justice.

These stylized facts and trends have recently raised the attention not only of legal scholars, who traditionally are deemed to exhibit a comparative advantage in these research fields, but also of economic theorists who have challenged their analytical apparatus in the area of litigation. The economic approach to litigation has stressed that the resort to plea bargains can be useful to preserve resources that would be dissipated at trial[4] and to improve the accuracy of the legal system. According to Grossman and Katz (1983), sanctions at pre-trial can be structured in order to provide a self-selection mechanism of the guilty defendant that contributes to the minimization of type I (conviction of an innocent) and type II (acquittal of a guilty) errors. Reinganum (1988) develops a model that allows to define the optimal level of prosecutorial discretion at pre-trial, showing that a regime that binds the prosecutor to a uniform plea bargain can be welfare improving if her prior in terms of probability of guilt is sufficiently high. In this case, most indictees are indeed punished, and the few innocent can self-select rejecting the plea offer, awaiting for a complete acquittal at trial.

In the existing literature, the welfare implications of the plea bargaining system and of restricted vs. unrestricted prosecutorial discretion at pre-trial hinge on an exogenous parameter: the probability of guilt (or proportion of guilty defendants in the population of the indictees) that is assumed to be known by the prosecutor by means of a learning process that is not specified. This hypothesis on the side of the prosecutor is particularly unsatisfactory since the proportion of guilty is the crucial variable in the administration of justice. If a prosecutor knew the probability of guilt of the defendant *ex ante*, the whole

[1] U.S. Department of Justice, Sourcebook of Criminal Statistics (1990).

[2] See White (1988).

[3] Civil law (Roman-Germanic) countries and Common Law (Anglo-Saxon) countries differ essentially in trial procedures. The former rely upon the inquisitorial role of the prosecutor, the latter resort to an *adversary system* in which the advocates of the litigating parties (one of which is the prosecutor acting on behalf of the state) are essentially free to bargain under the supervision of an impartial but passive judge. See Dewatripont, Tirole (1995).

[4] See Gould (1973). Landes (1971).

administration of justice would be radically simplified. All cases would end at pre-trial where the appropriate expected penalty would be imposed, nullifying the need of disclosure of effective liabilities through a costly trial.

From these premises, this work is intended to test the "rational expectations" hypothesis of the prosecutor trying to model explicitly the learning process that is assumed in the previous literature. In this direction, we consider a simple strategic interaction between prosecutor and defendant as a two-sided incomplete information game and define the condition under which the self-selection mechanism of the plea bargain procedure provides a learning device. If through a plea offer the prosecutor receives unbiased information concerning the true type of defendant, we claim that the informative strategy can be played in the repeated game to learn in a Bayesian fashion the true proportion of guilty in the whole sample of indictees. In our formalization trials are biased by random events; therefore the prosecutor cannot rely on trial outcomes to update her beliefs about the effective guilt of the opponent. Plea bargains are the only learning device available to the prosecutor.

Under these assumptions, we show that if the stochastic process that governs the evolution of guilty and innocent defendants exhibits some stationarity properties, a prosecutor who systematically resort to plea bargaining in the limit will learn the true proportion of guilty defendants in the population of indictees.

The paper is organized as follows; in section 2, 3 we model a criminal suit as a Bayesian game showing how the plea bargaining system provides, for some critical parameter configurations, a self-selection mechanism for the guilty defendant. Section 4 contains the analysis of the asymptotic behavior of the subjective probability of a prosecutor in the repeated game when the prosecutor systematically opts for the informative strategy. Section 5 concludes.

2 A litigation and settlement game

2.1 Notation

The strategic interaction between prosecutor and defendant is modeled as a game with two-sided incomplete information. Uncertainty materializes, according to Harsanyi (1968), in the *types* of players that differ in terms of private information. We are primarily concerned in modeling a criminal suit where a benevolent prosecutor endeavors to disclose the truth in terms of effective liabilities and to impose the sanction that society deems appropriate for a given misconduct; the defendant seeks to minimize the penalty imposed by the prosecutor.[5] The prosecutor P, after preliminary investigations and hearings, has

[5]For sake of realism, we should consider the interaction between the prosecutor and the legal counsel of the defendant. At first approximation, we assume that no agency problem emerges between the advocate and her client. For extensions, see Gilson and Mnookin (1995).

collected evidence of guilt or innocence that will be considered the private information of P. Formally, $P = \{\pi_g, \pi_i\}$, where π_g, π_i are signals that the prosecutor receives concerning the guilt or innocence of the opponent.[6] The defendant D knows to be guilty or innocent, so $D = \{G, I\}$.

The action space of the players is : $A_P = \{S, T\}$ and $A_D = \{A, R\}$ where S represents the choice to settle the case through a plea bargain and T to opt for a full trial; A and R refer to the acceptance or rejection of the plea offer by the defendant.

Players are risk neutral and play pure strategies: $s_P : P \rightarrow (A_P)$ and $s_D : D \rightarrow (A_D)$.

The probabilistic structure of the model is the following: $p \in [0, 1]$ is the prior of the prosecutor about the guilt of the defendant. Consider then the following matrices of joint probabilities:

$P(\pi\|\cdot)$	G	I
π_g	α	$1-\beta$
π_i	$1-\alpha$	β

$P(v\|\cdot)$	G	I
v_g	γ	$1-\delta$
v_i	$1-\gamma$	δ

where $\alpha, \beta, \gamma, \delta \in [0, 1]$. The first matrix contains summary information about the efficiency of pre-trial investigations; when α and β are high, the prosecutor is considerably confident of the reliability of the evidence of guilt or innocence available. In the second matrix, v_g and v_i represent the event of verdict of guilt or innocence rendered by the court; γ and δ therefore can be interpreted as parameters that indicate the efficiency of the judicial system. If δ is very low, type I errors often occur so that a large proportion of innocent defendants is convicted; if on the contrary γ is very low, many guilty defendants are found innocent at trial and type II errors are frequent. We assume that $\alpha, \beta, \gamma, \delta$ are common knowledge.[7]

[6] Some authors (Reinganum, 1988) define the private information of the prosecutor as $\pi \in [0, 1]$ as the *strength of the case*, a summary statistic of the extent and quality of the evidence available to her. The extension of the present formalization to encompass the continuum of types would not dramatically change our results.

[7] These assumptions deserve some comments; essentially in this model trial is a random device which convicts a defendant with a fixed probability ad acquits him with is complement to one. Our formalization recalls Rebelais' judge Bridoye who after laborious and time-consuming presentation of evidence invariably decides his cases by the fall of a dice. (*Gargantua et Pantagruel*, 3:39-40). This heuristic, paradoxical as it can seem, nevertheless takes explicitly into account the unavoidable randomness of the litigation process that the term "trial" itself evokes. The common knowledge on the parameters that feature the investigation and the administration of justice can be justified assuming that all players know that some innocent will be involved in the investigations and eventually punished just because society has committed itself to a given conviction rate to ensure deterrence. For the use of lotteries in social decision making see Elster (1988), Calabresi and Bobbit (1971).

The matrix of joint probabilities is given by:

$P(\pi, \cdot)$	C	I
π_c	$p\alpha$	$(1-p)(1-\beta)$
π_i	$p(1-\alpha)$	$(1-p)\beta$

from which we immediately compute the marginals, $P(\pi_c) = p\alpha + (1-p)(1-\beta)$ and $P(\pi_i) = p(1-\alpha) + (1-p)\beta$.

2.2 Pay-offs and objective functions

The payment structure of P and D are at most expressed in the form of lotteries. These lotteries are based for both players on the following Von Neumann-Morgestern utils ordering:

$$I_G < G_G < I_S < I_I = 0 < G_S < G_I \tag{1}$$

Notation is as follows: the first letter indicates the type of defendant, while the subscripts refer either to the verdict rendered by the court or to the acceptance of S by the defendant. For sake of clarity, I_S for instance represents the pay-off obtained by an innocent defendant when he accepts the settlement offer.[8] The guilty defendant who accepts the settlement offer is better off compared to the innocent acquitted. This entails that crime is profitable if the case is settled at pre-trial. We assume further that $|I_G| > |G_I|$, i.e. an individual level type I error is more serious than type II error.[9]

Let us turn to the expected pay-offs.

The defendant. When D pleads guilty and accepts S, his pay-off are read in (1); if the case is litigated, the expected pay-offs x_D and y_D for the guilty or innocent defendant are given by:

$$x_D = \gamma G_G + (1-\gamma) G_I \tag{2}$$

$$y_D = \delta I_I + (1-\delta) I_G \tag{3}$$

[8] Several assumptions are embodied in this pay-off ordering; the disutility accruing to an innocent defendant when he accepts a plea bargain is lower than the disutility in the case of a fair verdict of guilt. This may happen when the plea bargain is relatively "cheap" compared to the sanction that can be imposed after a full trial. We could nevertheless invert the second inequality to consider a variation of the pay-off structure that the advocate of civil liberties will find more palatable. We assume that neither psychic cost nor loss of reputation emerge from the acquittal of the innocent defendant.

[9] See Posner (1983).

The prosecutor. The objective function of P is given by a convex combination of D's pay-offs, weighted by the subjective probability of guilt. When D accepts S, P's expected pay-offs are given by:

$$z_P = pG_S + (1-p)I_S \tag{4}$$

$$w_P = pI_S + (1-p)G_S \tag{5}$$

The prosecutor enjoys z_P if the defendant is guilty.[10] The first term represents the expected settlement intake and the second the expected disutility from imposing a penalty, even at a discount rate, to an innocent defendant. The prosecutor collects instead w_P if the defendant is innocent; in this case plea bargaining becomes more profitable for P as long as the probability of guilt decreases. This pay-off structure is such that the utility of the prosecutor increases with the soundness of her belief.

The same logic is adopted to define the expected payment from trial; they are given by the following expressions:

$$x_P = \gamma[p(-G_G) + (1-p)I_G] + (1-\gamma)[pI_I + (1-p)(-G_I)] \tag{6}$$

$$y_P = \delta[p(-G_I) + (1-p)I_I] + (1-\delta)[pI_G + (1-p)(-G_G)] \tag{7}$$

In equations (6) and (7), x_p and y_p define the expected pay-offs in the case that the defendant is respectively guilty or innocent.[11] In figure 1, we plot x_p and y_P and then clarify their economic implications.

According to the values of the parameter that feature the judicial system, γ and δ, the expected pay-offs lie between the two lines in both graphs; it is worthy to notice that they are increasing with the soundness of the beliefs of the prosecutor exactly as it happened at pre-trial. In particular, x_p confirms the intuition that when the proportion of guilty

[10] It is worthy to stress that the fact the defendant is guilty obviously does not entail that $p = 1$, so that the subjective evaluation of the prosecutor conforms the truth; she is fully aware of the possibility of committing errors and this reflects on her expected pay-off.

[11] In order to construct appropriately the objective function of the prosecutor, we must invert the sign of some pay-off since the interest of litigating parties diverge in some respects. The sign of G_G has to be inverted since a fair verdict for a guilty defendant represents a loss at the individual level but a positive sign in the objective function of the prosecutor. Analogously, the acquittal of a guilty (G_I) entails the occurrence of a type II error for the prosecutor but the attainment of the maximum gain for the defendant. On the contrary as far as type I errors are concerned, they represent a loss at the social and at the individual level.

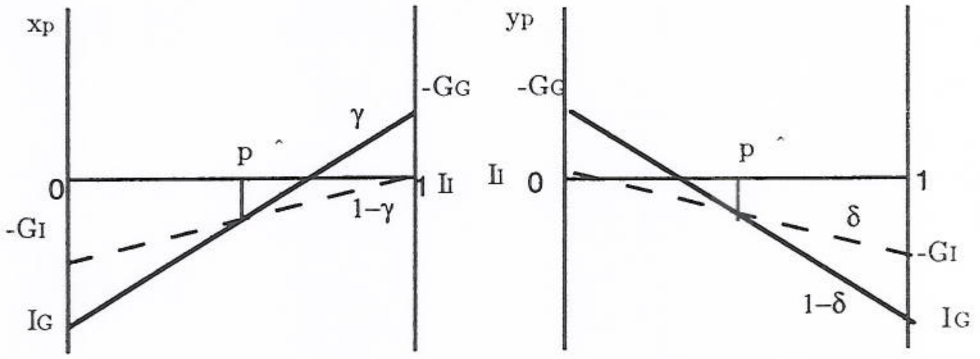

Figure 1: Expected payment of P when D is guilty (x_P) or innocent (y_P)

defendants is higher than the critical threshold $\hat{p}$ the prosecutor is better off when they are all convicted. The intuition behind y_p is less straightforward; when the proportion of innocent defendants is higher than the critical threshold, the prosecutor is better off when they are all convicted. In fact, this counterintuitive result is due to the assumptions about the payment scheme that yields for the prosecutor a higher pay-off when the guilty is convicted than when the innocent is acquitted. This differential in terms of pay-offs represents a premium for the additional inquisitorial effort that the prosecutor must display when the interest of litigating parties diverge.[12]

2.3 The game in extensive form

In figure 2, a *chance move* generates the two types of defendant, guilty or innocent.[13] Nature plays again to generate the types of prosecutor π_C, π_I according to the probability matrix previously described.[14] Two different information sets for the prosecutor exist, each corresponding to a type: since the prosecutor does not know the type of defendant, she just knows that given the evidence of guilty the defendant is guilty with probability α, and that given the evidence of innocence he is innocent with probability β. At each information set, P may play the strategy T, opting for litigation, or play S namely offer a fixed settlement offer in exchange of a guilty plea.[15] D knowing his guilt or innocence but

[12]By this token at common law the prosecutor must prove guilt "beyond any reasonable doubt".

[13]It is clear from this assumption that we are studying litigation and settlement in a framework of adverse selection; it could be possible to study the issue as a moral hazard problem in which uncertainty over the types emerges from the individual decision to commit or not the crime.

[14]In this model, investigation and pre-trial negotiations are separate stages of the enforcement of laws. The prosecutor is not encharged to contribute to the collection of evidence that is provided by the investigation agency. In fact, albeit many substantial differences both in the adversary and in the inquisitorial system the prosecutor is entitled to play an active role in investigation activities.

[15]In this model we are not concerned in the determination of the optimal settlement amount. This could be easily done attaching an expected utility function as a function of the settlement amount that could capture the fact that increasing the settlement the prosecutor raises the likelihood that the case will be

ignoring the strength of the case of P accepts or reject the plea bargain, respectively A or R. The bargaining procedure takes therefore the simple form of a take-it-or-leave-it offer. If negotiations break down, the case is litigated at trial where the parties collect the same pay-offs that are obtained if no pre-trial negotiation occurs.

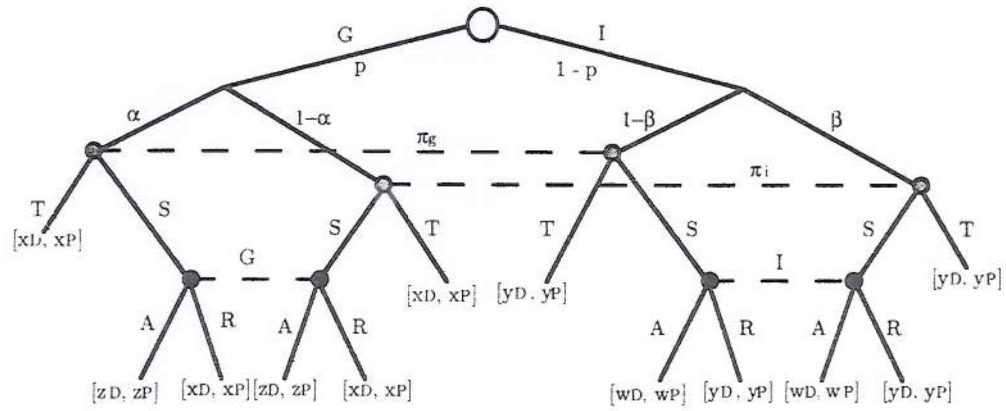

Figure 2: The game tree

3 Bayesian equilibria: an example

The solution concept we adopt for the litigation and settlement game is bayesian equilibrium.[16] We consider only pure strategies and we do not carry out a closed-form characterization but solve the game for the following pay-offs structure:

$$I_C = -6; C_C = -2; I_S - 1; I_I = 0; C_S = 1; C_I = 5$$

From the equation that define the expected pay-offs of P and D, we have:

$$[x_D, x_P] = [5 - 7\gamma, \gamma(8p - 6) + (1 - \gamma)(5p - 5)] \quad (8)$$

$$[y_D, y_P] = [6\delta - 6, \delta(-5p) + (1 - \delta)(2 - 8p)] \quad (9)$$

$$[z_D, z_P] = [1, 2p - 1] \quad (10)$$

$$[w_D, w_P] = [-1, 1 - 2p] \quad (11)$$

litigated. See Bebchuk (1984) and Nalebuff (1987).

[16] In simple terms, a bayesian equilibrium is a set of strategies and beliefs such that, at any stage of the game, strategies are optimal given the beliefs and the beliefs are obtained from equilibrium strategies and observed actions using Bayes' rule.

Assumption 1. $\alpha, \beta, \gamma, \delta > \frac{1}{2}$

By introducing this restriction on the "technological" parameters of the investigation and of the judiciary we explicitly reduce the randomness of the system: it is relatively more likely that the effective guilt and innocence emerge from the evidence at pre-trial and from the verdict rendered by the court.

3.1 Plea bargaining as a screening device

In this section we focus on the existence of a separating equilibrium that allows to screen perfectly the guilty from the innocent when the prosecutor plays a pooling strategy.

Proposition 1. *$[S, S], [A, R]$ is a bayesian equilibrium for $p < \bar{p}$ if and only if $\gamma \geq \frac{4}{7}, \delta \geq \frac{5}{6}$.*

PROOF. Let us suppose that $s_P(\pi_g) = s_P(\pi_i) = S$. By Bayes' rule, we get:

$$P(\pi_c|S) = \frac{P(S|\pi_c)P(\pi_c)}{P(S|\pi_c)P(\pi_c) + P(S|\pi_i)P(\pi_i)} = p\alpha + (1-p)(1-\beta) \equiv p^* \tag{12}$$

If inequality

$$\alpha p^*(1) + (1-\alpha)(1-p^*)(1) \geq \alpha p^*(5-7\gamma) + (1-\alpha)(1-p^*)(5-7\gamma) \tag{13}$$

is satisfied, $s_D(G) = A$ is a best reply for the guilty defendant. This is true when $\gamma \geq \frac{4}{7}$.

If

$$\beta(1-p^*)(-1) + (1-\beta)(p^*)(-1) < \beta(1-p^*)(6\delta-6) + (1-\beta)(p^*)(6\delta-6) \tag{14}$$

then $s_D(I) = R$ is a best response for the innocent. Inequality (14) holds for $\delta \geq \frac{5}{6}$.

Let us know check if given the best reply of D, a pooling plea bargain is an equilibrium strategy for P.

By Bayes' rule we get:

$$P(G|\pi_g) = \frac{\alpha p}{\alpha p + (1-\beta)(1-p)} \equiv \tilde{p} \tag{15}$$

When

$$\tilde{p}(2p-1) + (1-\tilde{p})[\delta(-5p) + (1-\delta)(2-8p)] \geq$$

$$\tilde{p}[\gamma(8p-6) + (1-\gamma)(5p-5)] + (1-\tilde{p})[\delta(-5p) + (1-\delta)(2-8p)] \tag{16}$$

holds, then $s_P(\pi_c) = s_P(\pi_i) = S$ is an equilibrium strategy for P. This is true when $p \leq \frac{4+\gamma}{3+3\gamma} \equiv \bar{p}$.[17]

The equilibrium described in Proposition 1 entails self-selection of the guilty. This eventuality occurs when the judicial system is particularly accurate and efficient; the virdicts rendered by the courts often punish the guilty (i.e. γ is high), then guilty defendants face harsh expected penalties from trial and are prone to accept plea bargains with increased frequency. If innocent defendants are often acquitted at trial (i.e. δ is high), then they are more optimistic about trial outcomes and tend to reject the settlement offers awaiting for an acquittal.[18]

As far as the prosecutor is concerned, the restriction on the beliefs stated in the equilibrium condition deserves some observation. First of all, it easy to check that

$$\lim_{\gamma \to \frac{4}{7}} \bar{p}(\gamma) = \frac{32}{33}; \lim_{\gamma \to 1} \bar{p}(\gamma) = \frac{5}{6}$$

The threshold probability as a function of γ takes values in the interval $[\frac{5}{6}; \frac{32}{33}]$; the pooling plea bargain represents therefore an equilibrium strategy for a vast region of the prior probability of the prosecutor. Furthermore the threshold probability is decreasing in γ. This indicates that as γ increases plea bargaining becomes relatively less profitable; trial is now attractive since it allows the prosecutor to maximize her penalty intake with a reduced probability of incurring in type II errors.

Not surprisingly, the game exhibits a large multiplicity of equilibria; in particular it can be shown that all possible pairs of strategies are equilibrium strategies for different parameter restrictions. It could be possible to restrict only to "more plausible" equilibria according to the literature on refinements. This analysis is not carried out firstly because we conjecture that the our equilibrium would survive such tests and more importantly because for the scope of our analysis it is sufficient to have shown the existence of a "black box" that for some parameter configurations works as a screening device. In what follows we will therefore assume that the interaction takes place in an economic environment where equilibrium strategies are $[A, R]$ and $[S, S]$, $[T, T]$.

[17] In the case of the prosecutor π_i, the relevant inequality is the same up to $P(G|\pi_i)$ which holds for the same parameter restriction established for the prosecutor π_g.

[18] Note that as far the defendant is concerned his strategies depend univocally from the "tecnological" parameters that feature our stylized juducial sistem and not from the private information of the prosecutor. Even if the analytical structure we set forth could be validly used to study the issue of strategic information trasmission raised by Shavell (1989), the attention is riveted upon plea bargain as a self-selection mechanism for the defendant.

4 Plea bargaining and Bayesian learning

4.1 The repeated game

Suppose now that the one shot interaction is repeated over time. The prosecutor plays her institutional role during the whole professional career facing a sequence of indictees. Therefore P is a long run player, namely a permanent institution encountering short run defendants D.[19] The institutional setting allows the prosecutor to display discretionality at pre-trial; she can opt for a full trial or for a settlement in exchange of a guilty plea. From the previous analysis, we are allowed to claim that for different priors both can be played as equilibrium strategies.

P plays $[S, S]$ or $[T, T]$ in discrete time at dates $t = 0, 1, 2, ...$ according to her prior probability p; her pay-offs depend upon these strategies and upon the state of the nature that occurs at each date. The state space is $\Theta = \{C, I\}$. P does not know with certainty the process governing the evolution of the states. Such process is represented by a Markov process with transition probabilities that will be specified afterwards.

Assumption 2. *The only true possible proportions of guilty defendants in the population of indictees are σ^h and σ^ℓ, with $\sigma^h > \sigma^\ell$ and $\sigma^h, \sigma^\ell \in [0, 1]$. These proportions represent the support of the prior of P.*

Formally, supp $p = \{\sigma^h, \sigma^\ell\}$ and

$$p_t = \begin{cases} \sigma^h & \text{wp} \quad q_t \\ \sigma^\ell & \text{wp} \quad (1 - q_t) \end{cases}$$

Assumption 2 entails that there are only two possible states of the proportion of the indictees which differ in terms of the number of guilty defendants. The true proportion is present only in one of these. The prosecutor obviously does not know the true state of the population; nevertheless she is able to attach a subjective evaluation to each of these. Such probability is q_t and can be recovered directly from p_t since σ^h and σ^ℓ are known.

The prosecutor has available an informative strategy that allows to learn the true type of defendant; such strategy consists in a pooling plea bargaining that will be accepted only by the guilty defendant. The determination of the true type of defendant in the one shot game allows P to update her beliefs in a Bayesian fashion. When the prosecutor opts for litigation, no learning occurs and the subjective probabilities remain stationary.

In the following pages we will study the asymptotic behaviour of the beliefs of the prosecutor, in order to verify the assumption of "rational expectations" that is often introduced

[19]See Fudenberg, Kreps and Maskin (1990).

in the literature. This assumption entails that the prosecutor has available a learning device to estimate precisely the true proportion of guilty defendants in the population of the indictees. In our model plea bargaining is that learning device.

4.2 The learning dynamics

The learning dynamics of P can be formalized as follows. According to the results in § 3.1, we claim that if $q_t < \bar{q}$ (i.e. $p_t < \bar{p}$), then $[S, S]$ is the optimal strategy for P. On the contrary, if $q_t \geq \bar{q}$, then $[T, T]$ is the optimal strategy. Therefore only if the former condition holds P can updates her beliefs. Formally,

$$q_{t+1} = \begin{cases} q_t & \text{if} \quad q_t > \bar{q} \\ p_t(\sigma = \sigma^\ell \text{ or } \sigma^h | G \text{ o } I) & \text{if} \quad q_t \leq \bar{q} \end{cases}$$

Posterior probabilities have the following expression respectively in the case that D accepts the plea bargain (i.e. D is guilty) and that D rejects it (i.e. D is innocent):

$$P(\sigma = \sigma_h | G) = \frac{\sigma_h q_t}{\sigma_h q_t + \sigma_l (1 - q_t)} \equiv g(q_t) \tag{17}$$

$$P(\sigma = \sigma_h | I) = \frac{(1 - \sigma_h) q_t}{(1 - \sigma_h) q_t + (1 - \sigma_l)(1 - q_t)} \equiv i(q_t) \tag{18}$$

The two functions defined in (17) and (18) represent the transition probabilities of the Markov process that governs the evolution of the states, namely the sequence of guilty or innocent defendants that the prosecutor encounters during her judicial activity. P updates her beliefs according to $g(q_t)$ or $i(q_t)$ if the defendant is found guilty or innocent respectively. As stated in assumption 2, D is guilty with probability $q^* = \sigma^h, \sigma^\ell$. The learning dynamics is sketched in figure 3.

4.3 The asymptotic behaviour

We now tackle the core of the problem. We study the limit case in which there is no absorbing state; the belief q_t is never stationary; the prosecutor deviates in some cases from her equilibrium strategy opting sistematically for the informative strategy even when her prior would command to litigate the case. This simplified case will be extremely useful to derive the main condition of convergence of her beliefs.

From the study of $g(q_t)$ and $i(q_t)$ it can be shown that the dynamical system of the beliefs is the one plotted in figure 4.

Suppose that at $t = 0$, P's belief is q_0. From figure 4, with probability q^*, q_1 will be given by the function $g(q_0)$; with probability $1 - q^*$, it will be given by $i(q_0)$.

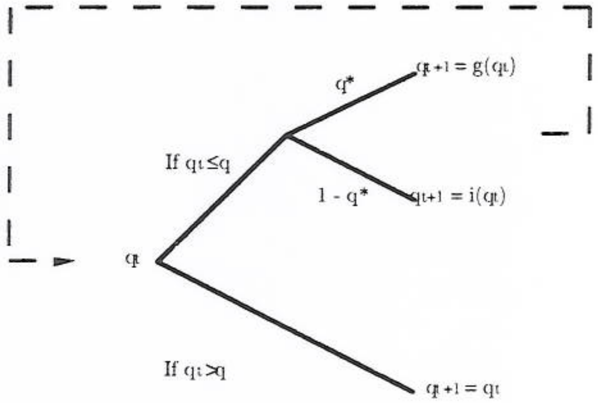

Figure 3: Inflow chart of P's beliefs

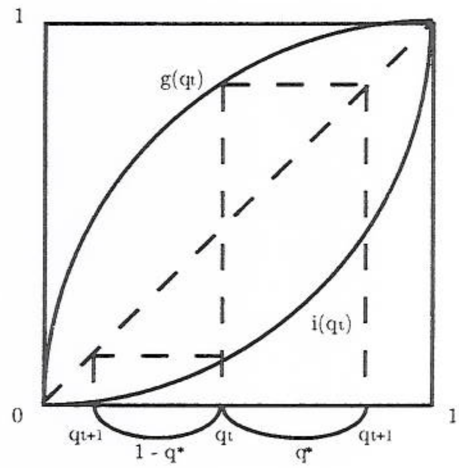

Figure 4: The dynamical system of P's beliefs

We now turn to the question of determining the asymptotic behaviour of q_t when is $q^* = \sigma^h$ or $q^* = \sigma^\ell$.

We introduce the following definitions:

Definition 1. *If* $q_t \overset{a.s.}{\to} 1$ *when* $q^* = \sigma^h$ *and if* $q_t \overset{a.s.}{\to} 0$ *when* $q^* = \sigma^\ell$ *then* P *learns from plea bargains.*

Consider the stochastic process of the weight on σ^h relative to σ^ℓ when the event G or I occurs. Defining $X_t(\binom{C}{I}) = \frac{q_t}{1-q_t}$, we get

$$X_{t+1} = \prod_{\tau=1}^{t} X_\tau X_0 \tag{19}$$

Taking logs and rearranging we have:

$$\frac{1}{t}\log X_{t+1} = \frac{1}{t}\sum_{\tau=1}^{t}\log X_\tau + \frac{1}{t}\log X_0 \tag{20}$$

Suppose X_t is a *i.i.d.* random variable; therefore from the Strong Law of Large Numbers, the mean of the logarithm of the random variable converges almost surely to the expected value, the constant μ. Formally,

$$\frac{1}{t}\sum_{\tau=1}^{t}\log X_\tau \xrightarrow{a.s.} E\log X_t = \mu$$

By substitution we have:

$$\frac{1}{t}\log\frac{q_{t+1}}{1-q_{t+1}} \xrightarrow{a.s.} \mu \tag{21}$$

Some manipulation yield:

$$\frac{q_{t+1}}{1-q_{t+1}} \xrightarrow{a.s.} e^{\mu t} \tag{22}$$

From (22) it is straightforward to see that if $\mu > 0$, then $q_{t+1} \xrightarrow{a.s.} 1$; otherwise if $\mu < 0$, then $q_{t+1} \xrightarrow{a.s.} 0$. The simple study of the sign of the expected value indicates if the proportion dominates the second or viceversa.

Let us apply the above mentioned result in our case considering the values of our transition probabilities. By Bayes rule, if the state is G, we have:

$$q_{t+1} = P(\sigma_h|G) = \frac{\sigma_h q_t}{\sigma_h q_t + \sigma_l(1-q_t)} \tag{23}$$

$$1-q_{t+1} = P(\sigma_l|G) = \frac{\sigma_l(1-q_t)}{\sigma_h q_t + \sigma_l(1-q_t)} \tag{24}$$

If the state is I, we get:

$$q_{t+1} = P(\sigma_h|I) = \frac{(1-\sigma_h)q_t}{(1-\sigma_h)q_t + (1-\sigma_l)(1-q_t)} \tag{25}$$

$$1-q_{t+1} = P(\sigma_l|I) = \frac{(1-\sigma_l)(1-q_t)}{(1-\sigma_h)q_t + (1-\sigma_l)(1-q_t)} \tag{26}$$

Let us build X_t as previously defined and firstly consider the case that the true proportion is the highest, σ^h. The variable X_{t+1} is given by the following expression:

$$X_{t+1} = \begin{cases} \frac{\sigma^h}{\sigma^\ell}\frac{q_t}{1-q_t} & \text{wp} \quad \sigma^h \\ \frac{1-\sigma^h}{1-\sigma^\ell}\frac{q_t}{1-q_t} & \text{wp} \quad 1-\sigma^h \end{cases}$$

The expected value of the random variable $\log X_{t+1}$ is given by:

$$\mu_1 = E[\log X_{t+1}] = \sigma^h \log \frac{\sigma^h}{\sigma^\ell} + (1-\sigma^h) \log \frac{1-\sigma^h}{1-\sigma^\ell} \tag{27}$$

Consider now the alternative case when the proportion of guilty is instead σ_l. The relevant expected value becomes

$$\mu_2 = E[\log X_{t+1}] = \sigma^\ell \log \frac{\sigma^h}{\sigma^\ell} + (1-\sigma^\ell) \log \frac{1-\sigma^h}{1-\sigma^\ell} \tag{28}$$

A short proof will show that μ_1 is positive and that μ_2 is negative.

PROOF. Consider the function

$$f(\alpha, x) \equiv \alpha \log x + (1-\alpha) \log(1-x)$$

. This function has a global maximum at $x = \alpha$, for all $\alpha \in [0, 1]$. So $f(\alpha, \alpha) - f(\alpha, x) > 0$ for all $x \neq \alpha$. Rearranging (27) we get

$$\mu_1 = \sigma^h \log \sigma^h + (1-\sigma^h) \log(1-\sigma^h) - [\sigma^h \log \sigma^\ell + (1-\sigma^h) \log(1-\sigma^\ell)]$$

which is positive.

By the same token, rearranging (28), we get

$$\mu_2 = \sigma^\ell \log \sigma^h + (1-\sigma^\ell) \log(1-\sigma^h) - [\sigma^\ell \log \sigma^\ell + (1-\sigma^\ell) \log(1-\sigma^\ell)]$$

which is negative. *Q.E.D.*

We are allowed to claim that given the positivity of μ_1 the belief q_t converges almost surely to one when the proportion of guilty is σ_h so that in the limit $p_t = \sigma_h$. On the contrary, given the negativity of μ_2 the belief q_t converges almost surely to zero when the proportion of guilty is σ_l so that in the limit $p_t = \sigma_l$.

Therefore we have established the following proposition:

Proposition 2. *P learns almost surely the true proportion of guilty defendants in the population of indictees when sistematically opts for the pooling plea bargaining* $[S, S]$.

5 Conclusions

This paper attempts to test theoretically an usual assumption of the strategic literature on litigation and settlement, namely that the prosecutor knows the true proportion of guilty defendants in the population of the indictees. Aim of the paper is to model explicitly a process of Bayesian learning for the prosecutor that hinges on the viability of the plea bargaining procedure as a screening device. In a simple two-sided incomplete information game, we show that the plea bargaining may work as a self-selection mechanism for the guilty defendant. Plea bargaining can therefore represent an informative strategy for the prosecutor to receive unbiased information about the type of opponent. Assuming that the plea bargaining procedure represents the only learning device since trial outcomes are stochastic, we show that under some condition of stationarity of the process governing the evolution of states a prosecutor who systematically opts for the informative strategy asymptotically learns the truth in terms of proportion of guilty defendants. This result confirms the achievements of the literature on rational learning that have shown that a bayesian player whose prior contains the truth will learn the truth.

The resort to plea bargaining can therefore be justified from a theoretical point of view not only on grounds of preservation of economic resources devoted to the judiciary and of enhanced accuracy in the administration of justice as the existing literature has shown, but also since it provides a reliable learning device. Nevertheless, the fact that our result holds in the limit and that relatively strong assumptions have been introduced to obtain it casts a suspicious eye on the positive implications of the model even though if contains some normative features that the previous literature did not consider yet.

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